Local dispersion insert for particle accelerators

Vladimir Visnjic
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
(Received 16 June 1994; revised manuscript received 29 August 1994)

The local dispersion insert is a novel focusing element for accelerators which produces local dispersion and β waves while leaving all other beam parameters unchanged. The obtained local dispersion wave can be effectively used to control the transition energy, the so-called γ_T jump of an accelerator, and the beam randomization (mixing) in accelerators with stochastic cooling. Since the induced tune shift is zero, the local dispersion insert is an extremely efficient cell, typically requiring only a small number of magnets. The local dispersion wave can be of either sign; therefore, one can obtain both positive and negative $\Delta\gamma_T$, whereas in the conventional schemes it is always positive. The paper contains three applications of the local dispersion insert: the transition jump in the Main Injector at Fermi National Accelerator Laboratory and in the Relativistic Heavy Ion Collider at Brookhaven National Laboratory and an asymmetric lattice for the Antiproton Debuncher at Fermi National Accelerator Laboratory.

PACS number(s): 29.20.Lq, 41.85.Lc, 42.15.Eq

I. INTRODUCTION

The local dispersion insert described in this paper is a novel focusing element for synchrotrons which produces local dispersion and β waves while leaving all other beam parameters unchanged. This focusing element can effectively be used in order to alter an accelerator's lattice for a variety of purposes, in particular:

(1) Adjusting the transition energy of an accelerator, in particular during the transition crossing [1];

(2) Varying the frequency-momentum parameter η between its minimal value optimal for the rf acceleration of the beam and the maximal value optimal for the stochastic cooling [2];

(3) Designing (or modifying existing) accelerators in such a way as to minimize the "bad" mixing and maximize the "good" mixing and, thus, improve the efficiency of the stochastic cooling of the beam. η can also be time varied as in (2). In other words, these two methods can be combined.

Section II contains a short review of the relevant theory. Section III describes the local dispersion insert. Sections IV, V, and VI contain three applications of the local dispersion insert, which illustrate the above points: the transition energy adjustments for the transition crossing in the Main Injector at Fermi National Accelerator Laboratory (Sec. IV) and in the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory (Sec. V), and a new asymmetric (with $\eta=0$ and $\eta>0$ regions) lattice for the Antiproton Debuncher at Fermi National Accelerator Laboratory (Sec. VI). The conclusions are summarized in Sec. VII.

II. THE THEORY

The time of travel of a particle of momentum $p_0+\Delta p$ between two points s_1 and s_2 in an accelerator is $\tau_0(s_1,s_2)+\Delta\tau(s_1,s_2)$, where

$$\frac{\Delta\tau(s_1,s_2)}{\tau_0(s_1,s_2)}=-\eta(s_1,s_2)\frac{\Delta p}{p_0} , \tag{1}$$

where $\tau_0(s_1,s_2)$ is the time of flight of a particle with momentum p_0 between the points s_1 and s_2. The momentum slip factor $\eta(s_1,s_2)$ depends on the dispersion function $D_x(s)$, the local radius of curvature $\rho(s)$ of the beam trajectory, and the Lorentz parameter γ of the beam

$$\eta(s_1,s_2)=\frac{1}{L_{s_1s_2}}\int_{s_1}^{s_2}\frac{D_x(s)}{\rho(s)}ds-\frac{1}{\gamma^2} , \tag{2}$$

where $L_{s_1s_2}$ is the arc length between the points s_1 and s_2. For the entire ring, η becomes the closed loop integral

$$\eta=\frac{1}{C}\oint_C\frac{D_x(s)}{\rho(s)}ds-\frac{1}{\gamma^2} . \tag{3}$$

The integral term is usually denoted $1/\gamma_T^2$ where γ_T (the transition γ) has the meaning of the beam energy at which η changes sign.

Equation (1) has important consequences that make η one of the most crucial parameters in an accelerator: the rf bucket area turns out to be proportional to $|\eta|^{-1/2}$, the synchrotron frequency to $|\eta|^{1/2}$, and the randomization ("mixing") of the beam between the points s_1 and s_2 is proportional to $\eta(s_1,s_2)$.

The local dispersion insert, described in Sec. III, produces the dispersion wave $\Delta D_x(s)$, which is nonzero inside a given interval (s_1,s_2) and zero everywhere else and thus gives rise to $\Delta\eta(s_1,s_2)$ while $\eta(s_2,s_1)$ remains unchanged. It is worth noting that $\Delta\eta(s_1,s_2)$ can be of either sign. Various scenarios are possible depending on what one wants to achieve—a transition jump, a zero mixing region, a mixing enhancement, or both in different regions of an accelerator.

Usually (for example, in the CERN Proton Synchrotron and Fermilab Booster) the η change has been

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achieved by creating a large global distortion of the dispersion function. This may lead to new problems, the most serious being a large increase of the maximal dispersion and, with it, dynamic aperture limitations and beam loss. A local change of the dispersion function has been proposed in Ref. [3]. Although it works in principle, the conditions imposed on the accelerator (at least $\frac{1}{4}$ betatron wavelength available space in zero dispersion straight sections and the phase advance per cell only $\pi/2$) are so restrictive that it cannot be used in any of the existing accelerators. In addition, only one half of the magnets needed actually change γ_T, the other half being necessary only to compensate the tune shifts caused by the "active" ones. In that sense, the efficiency of this scheme is only 50%.

By contrast, the local dispersion insert consists of lenses that are positioned π apart in betatron phase advance. Therefore, it can be used in essentially all modern accelerators, which are in the rule based on the lattices with either $\pi/2$ or $\pi/3$ phase advance per cell. In particular, since the desired γ_T jump can be achieved by locally *decreasing* the dispersion, the maximal dispersion of the accelerator remains unchanged. Therefore, the problems mentioned above are absent. Although the maximal β function is higher than in the global dispersion wave method, the beam size can, in fact, be decreased due to the decrease of the dispersion.

All the magnets of the local dispersion insert are active, thus, the efficiency of the cell is 100% and a smaller number of magnets is needed. For example, in Fermilab's Main Injector, γ_T can be changed by one unit with as little as two triplets of quadrupoles, while in the RHIC at Brookhaven National Laboratory, six triplets of quadrupoles are sufficient to change γ_T by one unit. In the Antiproton Debuncher, η can be changed in an asymmetric way, thus creating a lattice with zero bad mixing and enhanced good mixing.

III. LOCAL DISPERSION INSERT

The local dispersion insert and the corresponding lattice functions distortions are shown in Fig. 1. It consists of three lenses with the focusing strength ratio $-2{:}1{:}1$, π apart in the betatron phase. The net effect of the cell is the localized dispersion wave between the like sign lenses. There is also a localized β wave inside the cell, however, the local dispersion insert does not change the tunes of the machine.

The optical equations of the local dispersion insert can be obtained as follows. A small perturbation of the gradient of a quadrupole causes a horizontal and a vertical tune shift, a distortion of the horizontal and vertical β functions (β wave), and a distortion of the dispersion function (dispersion wave). In planar machines, there is only the horizontal dispersion wave. A local dispersion-adjusting cell must have zero tune shifts and no free β and dispersion wave escaping it.

The horizontal and vertical tune shifts caused by the small quadrupole perturbation $\Delta B'(s)$ are

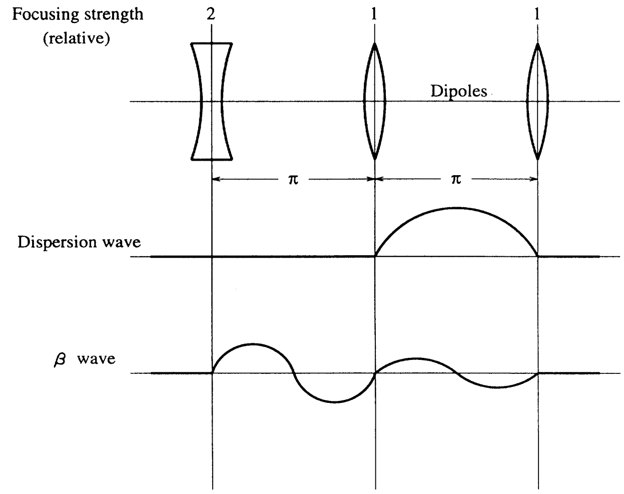

FIG. 1. The local dispersion insert and the corresponding dispersion and β waves.

$$\Delta\nu_x = \frac{1}{4\pi}\oint \beta_x(s)\frac{\Delta B'(s)}{B\rho}ds \ ,$$

and similar (of course, with the opposite sign) for $\Delta\nu_y$. For a thin quadrupole located at s_0,

$$\frac{\Delta B'(s)}{B\rho} = \Delta k\,\delta(s-s_0) \ ,$$

which gives

$$\Delta\nu_x = \frac{\Delta k \beta_x(s_0)}{4\pi} \ , \quad \text{and} \ \Delta\nu_y = -\frac{\Delta k \beta_y(s_0)}{4\pi} \ .$$

The perturbations of the β functions (β waves) downstream from the (thin) quadrupole are

$$\frac{\Delta\beta_x(s)}{\beta_x(s)} = -\Delta k \beta_x(s_0)\sin\{2[\mu_x(s)-\mu_x(s_0)]\}$$

and

$$\frac{\Delta\beta_y(s)}{\beta_y(s)} = \Delta k \beta_y(s_0)\sin\{2[\mu_y(s)-\mu_y(s_0)]\} \ .$$

The perturbation of the normalized dispersion function (dispersion wave) downstream from the quadrupole is

$$\frac{\Delta D_x(s)}{\sqrt{\beta_x(s)}} = -\Delta k D_x(s_0)\sqrt{\beta_x(s_0)}\sin[\mu_x(s)-\mu_x(s_0)] \ .$$

Notice that the β wave propagates with twice the frequency of the dispersion wave. No free dispersion wave will escape from a pair of quadrupoles of identical strength $(2n+1)\pi$ apart in phase, however, such an arrangement will produce β waves. Similarly, no free β wave will escape from a pair of quadrupoles of equal and opposite strengths $n\pi$ apart. Finally, no dispersion wave is created by a quadrupole in a zero dispersion region. By combining these three statements, we arrive at the local dispersion insert: Two quadrupoles of equal strength π apart in phase accompanied by another one with the double and opposite strength, π (or $n\pi$ if necessary) apart

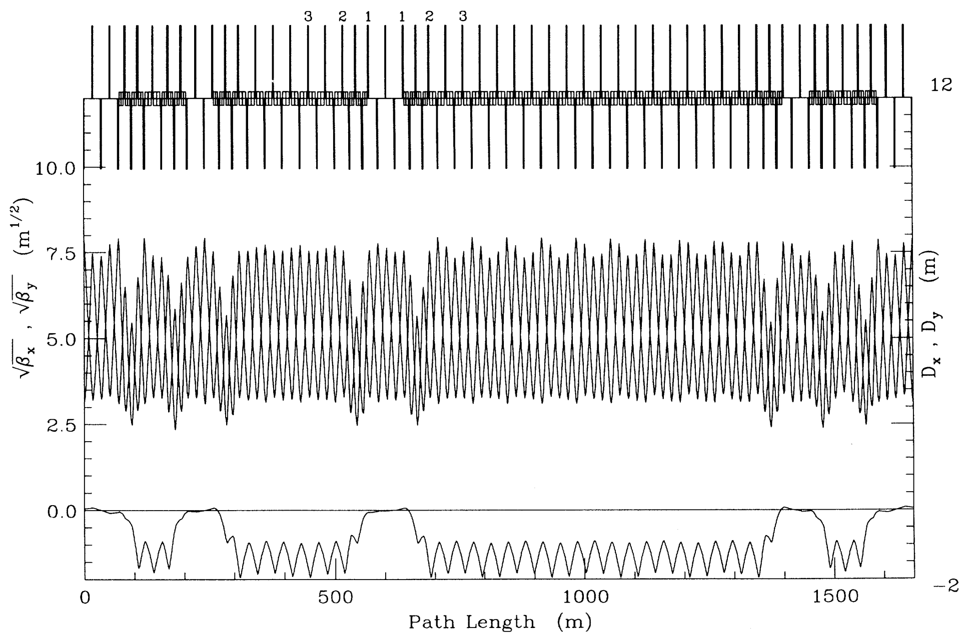

FIG. 2. The lattice functions in one superperiod of the Main Injector at Fermi National Accelerator Laboratory. $\gamma_T=21.59$.

and in a zero dispersion region. It is obvious from the above equations that the tune shifts are zero, the β wave is localized in the interior of the cell, and the dispersion wave is nonzero only between the like sign quadrupoles.

IV. LOCAL DISPERSION INSERT IN THE MAIN INJECTOR

The γ_T *jump,* which secures a speedy transition crossing, is probably the most widely known example of the need to change the value of η. The transition occurs at the energy $\gamma=\gamma_T$ and is characterized by a multitude of problems [1]. Rapid crossing of transition alleviates (or eliminates) these problems. In order to achieve the γ_T jump, the dispersion function must be temporarily changed such that $\dot{\eta}$ is chiefly determined by $\dot{\gamma}_T$ rather than $\dot{\gamma}$.

It has been widely acknowledged for several years now [4] that the transition crossing in the Main Injector is the main problem for this accelerator. The local dispersion insert is the solution of the problem.

The phase advance per cell in the Main Injector is $\pi/2$. The lattice functions of one half of the machine are shown in Fig. 2. See Ref. [5] for the various γ_T jump schemes proposed so far.

The natural positions for the local dispersion insert are

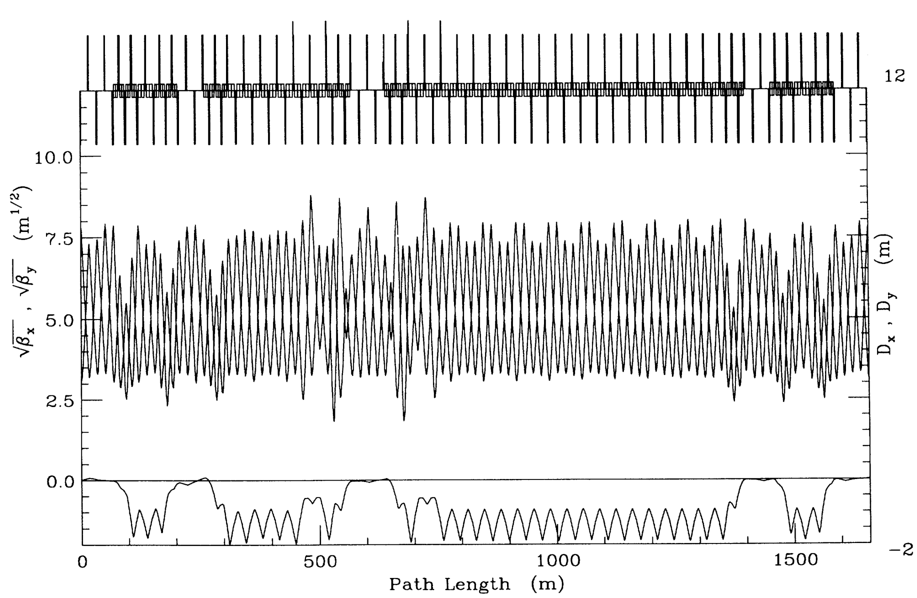

FIG. 3. The lattice functions in the Main Injector with the local dispersion insert. $\gamma_T=22.07$.

TABLE I. Relevant lattice parameters.

$\Delta\gamma_T$	Max. β_x (m)	Max. β_y (m)	Max. Dispersion (m)	$\Delta\nu_x$	$\Delta\nu_y$
0	59.1	62.7	1.95		
0.5	77.3	64.5	1.98	$<10^{-3}$	$<10^{-3}$

on each side of the two long arcs ending with zero dispersion regions. As an example, let us put two inserts to the left and right of the long straight section, at the quadrupoles marked by 1, 2, and 3, which are π apart (Fig. 2). The horizontal β function has maxima of the same size at these quadrupoles. In addition, the dispersion function is zero at the position of the quadrupole 1. Consequently, adding Δk to the quadrupoles 2 and 3 and $-2\Delta k$ to the quadrupole 1 will create a dispersion bump between the quadrupoles 2 and 3, and a β wave between the quadrupoles 1 and 3. There will be no changes in the remaining part of the ring.

Figure 3 illustrates the effect of the local dispersion insert well: The dispersion is increased in the inserts, while it is unchanged in the rest of the machine. As predicted, $\Delta\nu_x \approx \Delta\nu_y \approx 0$. Since $\Delta\gamma_T > 0$ is achieved by locally *decreasing* the dispersion, the maximal dispersion does not change.

Several relevant lattice parameters are shown in Table I for $\Delta\gamma_T$ of 0.5, achieved with the fractional gradient increases of approximately 12%.

In practice we need $\Delta\gamma_T \approx 1$, which can be achieved by adding one local dispersion insert on the other end of each of the two arcs. Due to the locality of the local dispersion insert, the values in Table I do not change except that $\Delta\gamma_T$ is doubled.

V. LOCAL DISPERSION INSERT IN THE RELATIVISTIC HEAVY ION COLLIDER

The lattice functions in one superperiod (one third) of the RHIC at Brookhaven National Laboratory are shown in Fig. 4. For a previous transition jump scheme, see Ref. [6]. The local dispersion insert works very well despite the fact that the phase advance per cell is only 80°, which has the consequence that the β and dispersion waves do not exactly cancel outside of it.

It is straightforward to install two local dispersion inserts in the first sextant. Figure 5 shows their effect for Δk of about 10% of the regular quadrupole strength: The dispersion is decreased in the inserts and only minimally affected in the rest of the machine. For example, the maximal dispersion increases by 24 cm to 2.08 m. The relevant beam parameters are shown in Table II.

VI. AN ASYMMETRIC LATTICE FOR THE ANTIPROTON DEBUNCHER

In machines with stochastic cooling [2], η is, as a rule, a compromise aimed at satisfying several different criteria. The radio-frequency operations (the beam capture, acceleration, rotation, etc.) require η to be minimized,

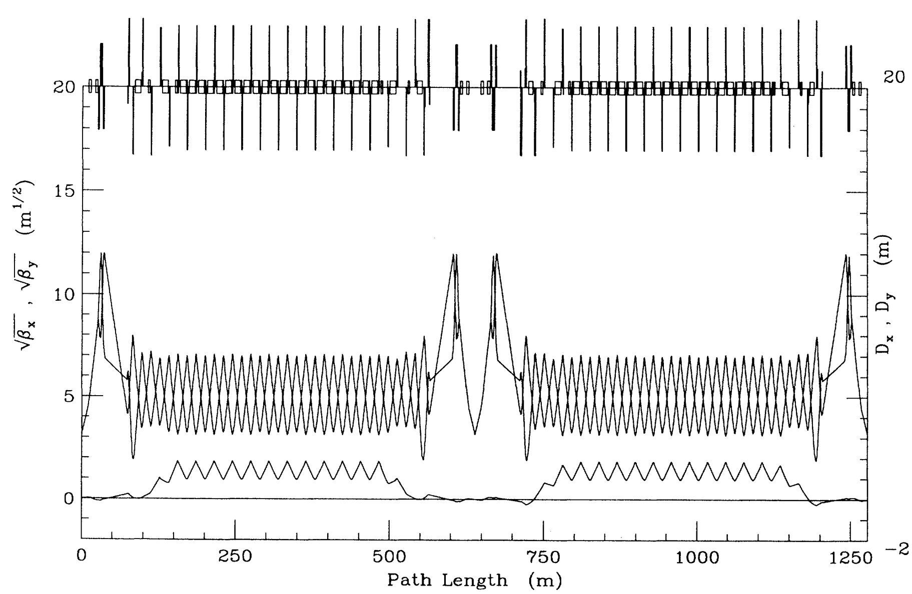

FIG. 4. The lattice functions in one superperiod of the Relativistic Heavy Ion Collider at Brookhaven National Laboratory. $\gamma_T = 22.9$ and the maximal value of the dispersion is 1.84 m.

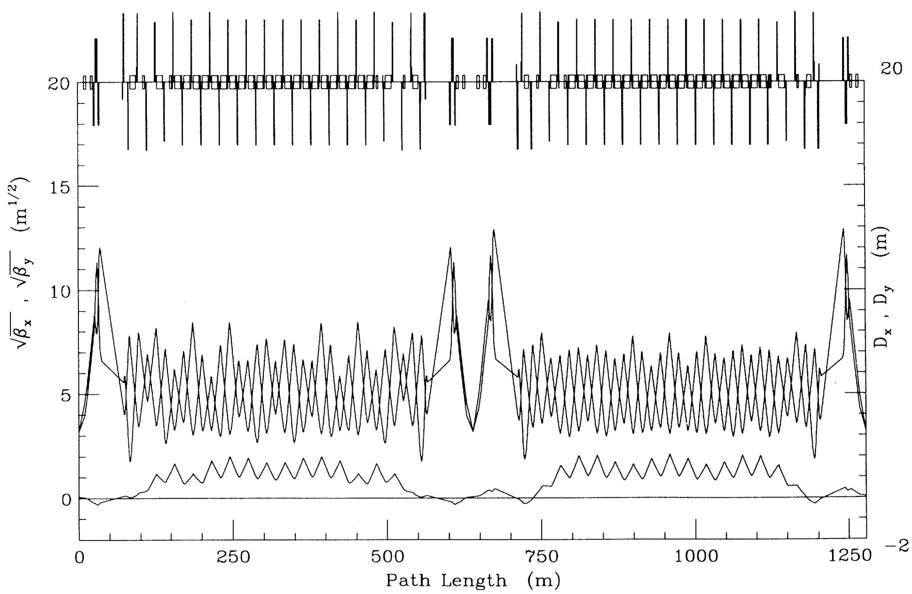

FIG. 5. The effect of the local dispersion insert in the RHIC lattice. γ_T is increased by half a unit to 23.4. The maximal value of the dispersion is 2.08 m. Due to imperfect cancellations of the β waves, the maximal β function increases from 144 to 166 m.

while the stochastic cooling requires η to be maximized such that the mixing factor [2]

$$M=\frac{p_0 f_0 \psi(p) \ln f_{\max}/f_{\min}}{2\eta WN} \tag{4}$$

is minimized. Here p_0 is the particle momentum, f its revolution frequency, $W=f_{\max}-f_{\min}$ the amplifier bandwidth, and N the number of particles in the beam. In addition, it is desirable to have a large value of η between the kicker and the pickup (good mixing) and $\eta=0$ between the pickup and the kicker (no bad mixing).

With the local dispersion insert it is possible to change η both locally and in time and thus realize an accelerator in which all of the above requirements are satisfied. Such a machine will have $\eta(PU,K)=0$ at all times, while $\eta(K,PU)$ will be varied during the operation between its minimal (for the radio-frequency acceleration of the beam) and the maximal (for the stochastic cooling) value. Machines with different values of η in different segments were discussed in the literature (see, e.g., Ref. [2]), however, the method presented here is the first one that (i) can be applied in an existing machine (as opposed to designing an asymmetric lattice from the start) and (ii) can be used during the operation of the accelerator in order to continuously optimize various regimes of the machine cycle. As an example, I present the design for the Fermilab Antiproton Debuncher, where by modifying a total of 18 quadrupoles the bad mixing is completely eliminated, while the good mixing can be varied between zero and twice or more its present value.

The superperiodicity of the machine is three, each superperiod consisting of two mirror-image halves. The phase advance per cell is $\pi/3$. The lattice functions of one sextant of the present machine are shown in Fig. 6. In order to achieve $\eta(PU,K)=0$ we need a *negative* local dispersion wave in the two sextants between the pickup and the kicker. The strength of the local dispersion insert needed is $(+0.16, +0.16, -0.32)$ in units of the regular quadrupole strength. In the remaining four sextants between the kicker and the pickup we need a *positive* local dispersion wave. The strength of the local dispersion insert needed in order to double the good mixing from its present value is $(-0.30, -0.30, +0.60)$ in the same units. The complete Debuncher lattice with η twice the present value and $\eta(PU,K)=0$ is shown in Fig. 7. This lattice is studied in detail in another publication [7]. We only note here that the stochastic cooling rate is roughly proportional to η and, therefore, also doubled. The maximal value of the (horizontal) β function increases with η, but quite substantial increases can be tolerated. For the lattice of Fig. 7 the beam size is still only one half of the initial beam size determined by the momentum spread and the maximal dispersion. Thus, there is no apparent reason for not aiming for even higher values of η.

TABLE II. Relevant beam parameters.

$\Delta\gamma_T$	Max. β_x (m)	Max. β_y (m)	Max. Dispersion (m)	$\Delta\nu_x$	$\Delta\nu_y$
0	144	143.7	1.84		
0.5	166.5	145.5	2.08	$<10^{-3}$	$<10^{-3}$

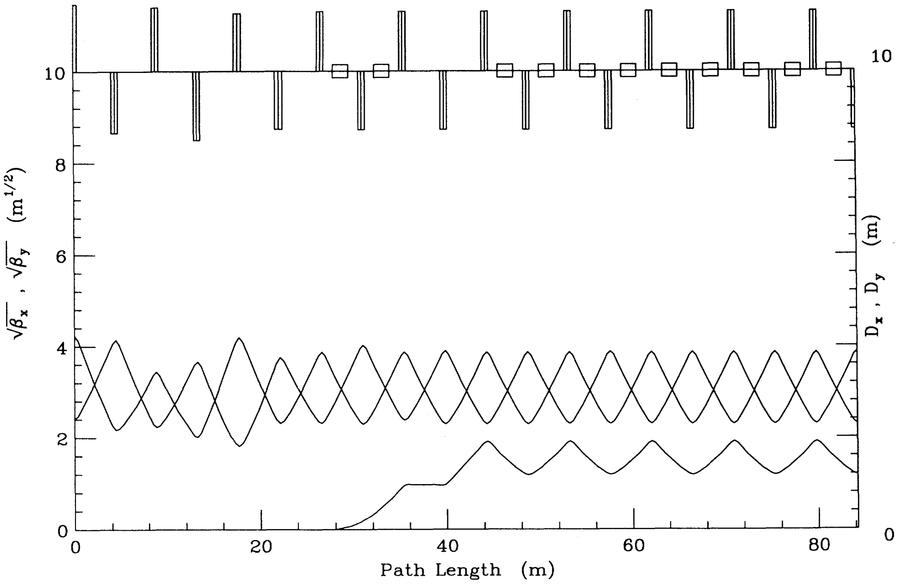

FIG. 6. The lattice functions in one superperiod of the Antiproton Debuncher at Fermi National Accelerator Laboratory.

VII. CONCLUSIONS

The local dispersion insert is a simple and efficient tool for the local adjustment of the dispersion function and, thus, the transition energy and the mixing in an accelerator. It is the only such cell proposed so far that can actually be used in real (existing or under construction) accelerators. In order for it to be used in an accelerator, the latter must have the phase advance per cell π/(integer) and at least one point of zero dispersion. Since the modern accelerator lattices are invariably based on either $\pi/2$ or $\pi/3$ phase advance per cell and have (because of that) zero-dispersion straight sections, the conditions for the application of the local dispersion insert are automatically fulfilled. How this works has been shown here on the example of three accelerators: the Main Injector and Antiproton Debuncher at Fermilab, and the Relativistic Heavy Ion Collider at Brookhaven National Laboratory.

It is worth pointing out that the local dispersion insert does not cause any undesirable side effects in addition to

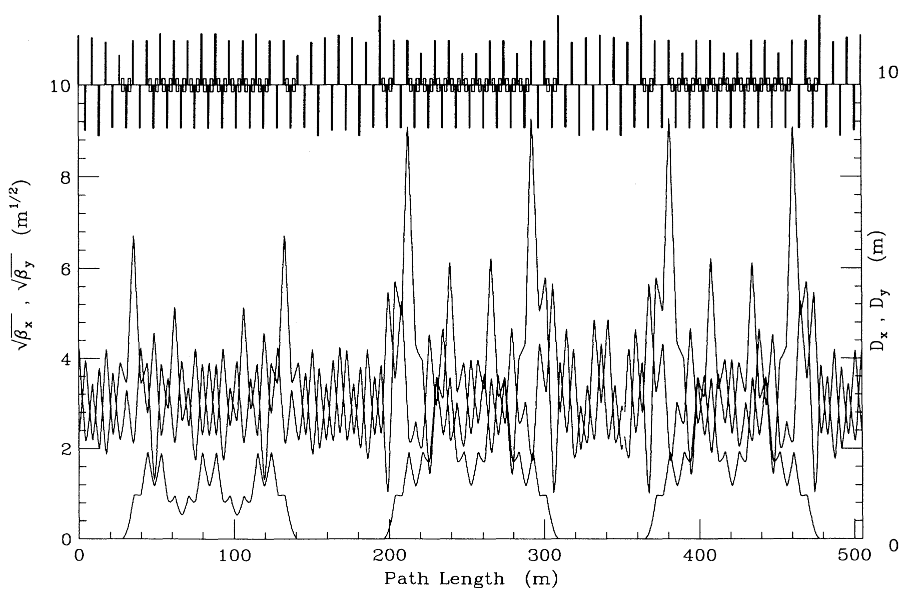

FIG. 7. The complete Debuncher lattice with zero bad mixing and η twice the value of the nominal lattice.

its intended effect, the change of the transition energy or the amount of mixing: It changes the dispersion locally, while leaving the global properties of the machine unaffected. In particular, the tunes of the machine and the maximal value of the dispersion remain unchanged.

ACKNOWLEDGMENTS

I would like to thank C. Ankenbrandt and D. Trbojevic for useful discussions and D. Edwards for helpful comments.

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